Title	Computing Mathematics – Introduction to Number Bases
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Overview of Binary Number System:

- Refers to a form of number where there are only 2 symbols representing numbers either a "0" or a "1"
- Used by computers to transmit, store, and encode data at a fundamental level

Overview of Octal Number System:

- Refers to a form of number where there are 8 symbols representing numbers, from '0', '1', '2', '3', '4', '5','6', '7'
- Used by people mostly to represent large binary numbers

Overview of Decimal Number System

- Refers to a form of number where there are 10 symbols representing numbers from 0 to 9
- Used by humans to represent the concept of numbers

Overview of Hexadecimal Number Systems

- Refers to a form of number where there are 16 symbols representing numbers, '0', '1', '2', '3', '4', '5' '6', '7', '8', '9','A', 'B', 'C', 'D', 'E','F'
- Used by people mostly to represent large binary numbers

Notation to prevent confusion in Computing Mathematics given below
In order to prevent students, programmers or technical staffs from experiencing confusion, here are some notations used concerning the arithmetic of binary, octal, decimal and hexadecimal.

Number System	Notation Used
Binary	10001_2 The number "10001" is a binary number and the subscript represent base '2', therefore, the concept of binary number
Octal	40731_8 The number 40731 is an octal number, and the subscript represent base '8', therefore, the concept of an octal number
Decimal	19331_{10} The number 19331 is a decimal number and the subscript represent base '10', therefore the concept of a decimal number

Hexadecimal	$17DFC_{16}$ The number 17DFC is a hexadecimal and the subscript represent base “16”, therefore the concept of a hexadecimal number

As you can see from the above 4 examples, each number is followed by a subscript that represents its base (also sometimes called radix).

Lookup table for binary, octal, decimal, and hexadecimal

Binary	Octal	Decimal	Hexadecimal
0	0	0	0
1	1	1	1
10	2	2	2
11	3	3	3
100	4	4	4
101	5	5	5
110	6	6	6
111	7	7	7
1000	10	8	8
1001	11	9	9
1010	12	10	A
1011	13	11	B
1100	14	12	C
1101	15	13	D
1110	16	14	E
1111	17	15	F
10000	20	16	10

Converting a decimal number to binary number via successive division method

Example 1.1

Convert 72_{10} into its binary counterpart

Dividend	Divisor	Remainder
72	2	0
36	2	0
18	2	0
9	2	1
4	2	0
2	2	0
1	2	1

The remainder numbers on the far right read from bottom to top is the number in binary counterpart, hence

$72_{10} = 1001000_2$

Converting a decimal number to octal via successive division method

Example 1.2

Convert 91_{10} into its octal counterpart

Dividend	Divisor	Remainder
91	8	3
11	8	3
8	8	1

Reading the numbers on the remainder column from bottom to top, we get 133_8

Converting a decimal number to hexadecimal via successive division method

Example 1.3

Convert 135_{10} into hexadecimal

Dividend	Divisor	Remainder
135	16	7
8	16	8

Reading the numbers on the remainder column from bottom to top, we get 87_{16}

Example 1.4

Convert 140_{16} into hexadecimal

Dividend	Divisor	Remainder
140	16	12 = C in hexadecimal
8	16	8

Hence, reading the remainder from bottom to top, we get $8C_{16}$

Conversion of Binary to Decimal Using Place Value Addition Method

Example 2.1

Convert 101_2 to into its decimal counterpart.

101_2

1	0	1
$\times 2^2$	$\times 2^1$	$\times 2^0$

Therefore $101_2 = 1(2^2) + 0(2^1) + 1(2^0) = 5_{10}$

Conversion of Binary to Octal and Hexadecimal Using Grouping Method
Example 2.2
Convert 1100110_2 into its Octal and Hexadecimal Counterparts

For Octal Conversion, we need to group the binary digits into groups of 3 bit
So, 1100110 is split up into the following (Split process has to be from right to left)
001 100 110

Each 3 bit correspond to a particular octal value in the table on the right below, by assigning every 3 bits to its octal equivalent, we can get the value in octal.

Please memorize the whole table if you are reading this document for a test or exam, as in exam or test environment, the table may not even be given to you.

Binary	Octal Equivalent
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Hence $1100110_2 = 146_8$

Example 2.3
Convert 11110111000 into its hexadecimal equivalent

Please memorize the whole table if you are reading this document for a test or exam, as in exam or test environment, the table may not even be given to you.

Binary	Hexadecimal Equivalent
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Split the number into 4 bits values and read from the lookup table
11110111000 can be split into the following 4-bit group from right to left

$111\ 1011\ 1000 = 78B_{16}$

Converting Octal & Hexadecimal to Decimal using Place Value Addition Method

Example 3.1

Convert 70152_8 into its decimal equivalent

70152_8

7	0	1	5	2
$\times 8^4$	$\times 8^3$	$\times 8^2$	$\times 8^1$	$\times 8^0$
28672	0	64	40	2

Decimal equivalent $= 28672 + 0 + 64 + 40 + 2 = 28778$

Example 3.2

Convert $2EF5_{16}$ into its decimal equivalent

2	E	F	5
× 16^3	14 × 16^2	15 × 16^1	× 16^0
8192	3584	240	5

8192+3584+240+5 = 12021_{10}

Convert from Octal & Hexadecimal to Binary through Binary Grouping Method

Binary	Octal Equivalent
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Example 4.1: Convert 317_8 into Binary
From the look up table on the right above and assigning each octal digit with its binary counterpart, we can deduce that the value $317_8 = 011\ 001\ 111_8$

Binary	Hexadecimal Equivalent
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Example 4.2: Convert $FE80_{16}$ into Binary
From the above table, we can assign each hexadecimal digit to its binary counterpart and deduce $FE80_{16} = 1111\ 1110\ 1000\ 0000$

Converting from Hexadecimal to Octal and Vice-Versa through Indirect Methods
Example 5.1: Convert $C3D2_{16}$ into octal

Firstly, we shall convert the number into binary and then using the grouping method and assigning of octal digit counterpart.

$C3D2_{16} = 1100\ 0011\ 1101\ 0010_2$

After converting the number into binary, we take a step further to convert the number into octal by grouping of 3-bit values from right to left

$1\ 100\ 001\ 111\ 010\ 010$

Assigning every 3-bit with its octal counterpart, we get the following

141722_8

Example 5.2: Convert 6315_8 into hexadecimal

Firstly, we need to convert the octal value 6315 into binary and then using the grouping method and assigning of hexadecimal digit counterpart.

$\mathbf{6315_8} = 110\ 011\ 001\ 101_2$

Once we have the value, we regroup the value into 4 bits and get the following basically

$1100\ 1100\ 1101_2$

And we can convert the value into hexadecimal which is CCD_{16}